AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TARGET JEE MATHEMATICS SOLUTION 13 OCTOBER 2019

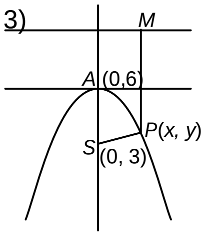
61. (b) The equation of the parabola referred to its vertex as the origin is $X^2 = lY$, where $x = X + a,\ y = Y + b$. Therefore the equation of the parabola referred to the point (a,b) as the vertex is

$(x-a)^2 = l(y-b)$ or $(x-a)^2 = \frac{l}{2}(2y-2b)$.

62. (c) $y^2 - 4y + 4 = 5x + 5 \Rightarrow (y-2)^2 = 5(x+1)$

Obviously, latus rectum is 5.

63. (a) Here vertex $\equiv (0, 6)$ and focus $\equiv (0, 3)$



then $Z \equiv (0,9)$ *i.e.*, $y = 9$

$\therefore$ Equation of parabola, $SP = PM$

$\Rightarrow \sqrt{(x-0)^2 + (y-3)^2} = |y-9|$

$\Rightarrow x^2 + y^2 - 6y + 9 = y^2 - 18y + 81$

or $x^2 + 12y = 72$.

64. (d) Let point of contact be (h, k), then tangent at this point is $ky = x + h$.

$x - ky + h = 0 \equiv 18x - 6y + 1 = 0$

or $\frac{1}{18} = \frac{k}{6} = \frac{h}{1}$ or $k = \frac{1}{3},\ h = \frac{1}{18}$.

65. (b) $S_1 \equiv x^2 - 108y = 0$

$T \equiv xx_1 - 2a(y + y_1) = 0 \Rightarrow xx_1 - 54\left(y + \frac{x_1^2}{108}\right) = 0$

$S_2 \equiv y^2 - 32x = 0$

$T \equiv yy_2 - 2a(x + x_2) = 0 \Rightarrow yy_2 - 16\left(x + \frac{y_2^2}{32}\right) = 0$

$\therefore \frac{x_1}{16} = \frac{54}{y_2} = \frac{-x_1^2}{y_2^2} = r \Rightarrow x_1 = 16r$ and $y_2 = \frac{54}{r}$

$\therefore \frac{-(16r)^2}{(54/r)^2} = r \Rightarrow r = -\frac{9}{4}$

$x_1 = -36,\ y_2 = -24,\ y_1 = \frac{(36)^2}{108} = 12,\ x_2 = 18$.

$\therefore$ Equation of common tangent

$(y - 12) = \frac{-36}{54}(x + 36) \Rightarrow 2x + 3y + 36 = 0$

Aliter : Using direct formula of common tangent $yb^{1/3} + xa^{1/3} + (ab)^{2/3} = 0$, where $a = 8$ and $b = 27$. Hence the required tangent is $3y + 2x + 36 = 0$.

66. (c) $y = -\frac{l}{m}x - \frac{n}{m}$

Condition for above line to be tangent to $y^2 = 4ax$ is $-\frac{n}{m} = \frac{am}{-l}$ or $nl = am^2$.

67. (b) Equation of parabola is $Y^2 = 4X$, where $X = x + \frac{5}{4}$

Tangent parallel to $Y = 2X + 7$ is $Y = 2X + \frac{a}{m}$

$\Rightarrow y = 2\left(x + \frac{5}{4}\right) + \frac{1}{2} \Rightarrow y = 2x + 3$ *i.e.*, $2x - y + 3 = 0$.

68. (b) $y = 2x + \lambda$ does not meet, if $\lambda > \frac{a}{m} = \frac{1}{2.2} = \frac{1}{4} \Rightarrow \lambda > \frac{1}{4}$.

69. (a) Tangent to parabola is, $y = mx + \frac{a}{m}$ .....(i)

A line perpendicular to tangent and passing from focus $(a, 0)$ is, $y = -\frac{x}{m} + \frac{a}{m}$ .....(ii)

Solving both lines (i) and (ii) $\Rightarrow x = 0$.

70. (c) $m_1 = \tan 45° = 1$, $m_2 = 3$

Slope of tangent $= \frac{3 \pm 1}{1 \mp 3} = -2$ or $\frac{1}{2}$

Tangent is $y = -2x + \frac{2}{-2}$ or $2x + y + 1 = 0$.

71. (a) Any line through origin is $y = mx$. Since it is a tangent to $y^2 = 4a(x - a)$, it will cut it in two coincident points.

$\therefore$ Roots of $m^2x^2 - 4ax + 4a^2 = 0$ are equal.

$\therefore 16a^2 - 16a^2m^2 = 0$ or $m^2 = 1$ or $m = 1, -1$

Product of slopes $= -1$. Hence it is a right angled triangle.

72. (b) Let the co-ordinates of P and Q be $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ respectively. Then $y_1 = 2at_1$ and $y_2 = 2at_2$. The co-ordinates of the point of intersection of the tangents at P and Q are $\{at_1t_2, a(t_1 + t_2)\}$

$\therefore y_3 = a(t_1 + t_2)$

$\Rightarrow y_3 = \frac{y_1 + y_2}{2} \Rightarrow y_1, y_3$ and y_2 are in A.P.

73. (c) $\because$ Parabola passes through the point (1,–2), then $4 = 4a \Rightarrow a = 1$

Formula for tangent, $yy_1 = 2a(x + x_1) \Rightarrow -2y = 2(x + 1)$

Required tangent is, $x + y + 1 = 0$.

74. (b) The equation of the tangent at point $(a, 2a)$ of the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$

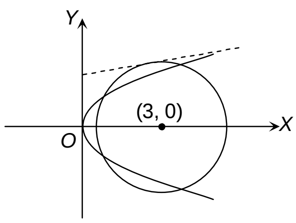
$\Rightarrow 2ay = 2a(x + a) \Rightarrow y = x + a$

This line makes an angle of $\pi/4$ with the x-axis, as $m = \tan\theta = 1$.

75. (c) Any tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$. It touches the circle, if $3 = \left|\frac{3m + \frac{1}{m}}{\sqrt{1 + m^2}}\right|$

or $9(1 + m^2) = \left(3m + \frac{1}{m}\right)^2$

or $\frac{1}{m^2} = 3$, $\therefore m = \pm\frac{1}{\sqrt{3}}$.



For the common tangent to be above the x-axis, $m = \frac{1}{\sqrt{3}}$

$\therefore$ Common tangent is, $y = \frac{1}{\sqrt{3}}x + \sqrt{3} \Rightarrow \sqrt{3}y = x + 3$.

76. (b) Any tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$

Since it passes throguh (1, 4), we have $4 = m + \frac{1}{m}$

$\Rightarrow m^2 - 4m + 1 = 0 \Rightarrow m_1 + m_2 = 4$, $m_1 m_2 = 1$

$\Rightarrow |m_1 - m_2| = 2\sqrt{3}$

If θ is the required angle, then $\tan\theta = \frac{2\sqrt{3}}{1+1} = \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{3}$.

77. (b) Any line through origin (0,0) is $y = mx$. It intersects $y^2 = 4ax$ in $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

Mid point of the chord is $\left(\frac{2a}{m^2}, \frac{2a}{m}\right)$

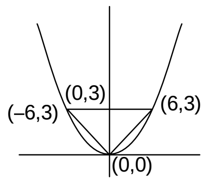
$x = \frac{2a}{m^2}$, $y = \frac{2a}{m} \Rightarrow \frac{2a}{x} = \frac{4a^2}{y^2}$ or $y^2 = 2ax$, which is a parabola.

78. (c) Equation of chord of contact of tangent drawn from a point (x_1, y_1) to parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$ so that $5y = 2 \times 2(x+2) \Rightarrow 5y = 4x + 8$. Point of intersection of chord of contact with parabola $y^2 = 8x$ are $\left(\frac{1}{2}, 2\right), (8, 8)$, so that length $= \frac{3}{2}\sqrt{41}$.

79. (a) The combined equation of the lines joining the vertex to the points of intersection of the line $lx + my + n = 0$ and the parabola $y^2 = 4ax$, is

$y^2 = 4ax\left(\frac{lx + my}{-n}\right)$ or $4alx^2 + 4amxy + ny^2 = 0$

80. (c) $\Delta = \frac{1}{2}(12 \times 3) = 18$ *sq. unit*



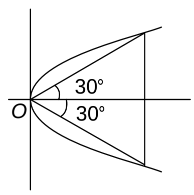
81. (b) $L_1 = \sqrt{3}y - x = 0$, solving L_1

and $S_1 \equiv y^2 - 4ax = 0$

Then $y = 4a\sqrt{3}$ and $x = 12a$

Hence $L = \sqrt{144a^2 + 48a^2}$

$= a\sqrt{192} = 8a\sqrt{3}$.



82. (b) Chord of contact of $(-1, 2)$ is $yy_1 = 2a(x + x_1)$ or $y = x - 1$.

83. (c) Equation of tangent at (1, 7) to $y = x^2 + 6$

$\frac{1}{2}(y + 7) = x.1 + 6 \Rightarrow y = 2x + 5$ …..(i)

This tangent also touches the circle

$x^2 + y^2 + 16x + 12y + c = 0$ …..(ii)

Now solving (i) and (ii), we get

$\Rightarrow x^2 + (2x+5)^2 + 16x + 12(2x+5) + c = 0$

$\Rightarrow 5x^2 + 60x + 85 + c = 0$

Since, roots are equal so

$b^2 - 4ac = 0 \Rightarrow (60)^2 - 4 \times S \times (85 + c) = 0$

$\Rightarrow 85 + c = 180 \Rightarrow 5x^2 + 60x + 180 = 0$

$\Rightarrow x = -\frac{60}{10} = -6 \Rightarrow y = -7$

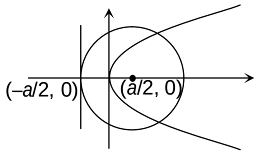
84. (c) Given parabola is $y^2 = 2ax$

$\therefore$ Focus (a/2, 0) and directrix is given by $x = -a/2$,

as circle touches the directrix.

$\therefore$ Radius of circle = distance from the point (a/2, 0) to the line

$(x = -a/2) = \frac{\left|\frac{a}{2} + \frac{a}{2}\right|}{\sqrt{1}} = a$



$\therefore$ Equation of circle be $\left(x - \frac{a}{2}\right)^2 + y^2 = a^2$ .....(i)

also $y^2 = 2ax$ .....(ii)

Solving (i) and (ii) we get $x = \frac{a}{2}, -\frac{3a}{2}$

Putting these values in $y^2 = 2ax$ we get

$y = \pm a$ and $x = -3a/2$ gives imaginary values of y.

$\therefore$ Required points are $(a/2, \pm a)$.

85. (b) Let point be (h, k). Normal is $y - k = \frac{-k}{4}(x - h)$ or $-kx - 4y + kh + 4k = 0$

Gradient $= -\frac{k}{4} = \frac{1}{2} \Rightarrow k = -2$

Substituting (h, k) and $k = -2$, we get $h = \frac{1}{2}$

Hence point is $\left(\frac{1}{2}, -2\right)$.

Trick : Here only point $\left(\frac{1}{2}, -2\right)$ satisfies the parabola $y^2 = 8x$.

86. (a) Normal at (h, k) to the parabola $y^2 = 8x$ is

$y - k = -\frac{k}{4}(x - h)$

Gradient $= \tan 60^\circ = \sqrt{3} = -\frac{k}{4} \Rightarrow k = -4\sqrt{3}$ and $h = 6$

Hence required point is $(6, -4\sqrt{3})$.

87. (d) $y = -2x - k$ is normal to $y^2 = -8x$

or $-k = -\{-4(-2) - 2(-2)^3\} = -(8 + 16) \Rightarrow k = 24$.

88. (d) We know that $t_2 = -t_1 - \frac{2}{t_1}$

Put $t_1 = 1$ and $t_2 = t$. Hence $t = -3$.

89. (c) Since the semi-latus rectum of a parabola is the harmonic mean between the segments of any focal chord of a parabola, therefore SP, 4, SQ are in H.P.

$$\Rightarrow 4 = 2.\frac{SP.SQ}{SP + SQ} \Rightarrow 4 = \frac{2(6)(SQ)}{6 + SQ} \Rightarrow SQ = 3.$$

90. (d) The equation of a normal to $y^2 = 4x$ at $(m^2, -2m)$ is $y = mx - 2m - m^3$. If the normal makes equal angles with the coordinates axes, then $m = \tan\frac{\pi}{4} = 1$. Thus, the required point is (1, – 2).